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The Passage of Fast Electrons Through Matter: The Work at the Radiation Center of Osaka Prefecture and Related Topics[1]

Second Revised Edition[2]

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The Passage of Fast Electrons Through Matter: The Work at the Radiation Center of Osaka Prefecture and Related Topics
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The first edition was dedicated to Professor Shigeru Okabe
on his sixtieth birthday.

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ABSTRACT

The experimental and computational work on the passage of fast electrons through matter done at the Radiation Center of Osaka Prefecture (RCOP) is reviewed. Related topics and problems to be solved are also described. The experimental work treated is the measurement of backscattering coefficients and charge-deposition distributions. Possible causes of discrepancies between experimental and Monte Carlo results are discussed. Among the results of the computational work, an empirical formula for the backscattering coefficient, a semiempirical formula for the extrapolated range and the algorithms for depth–dose distributions are described. In Appendix, the work on interpolation formulas for the continuous slowing-down approximation (CSDA) range is reviewed, and a new formula is suggested.[3]

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[3] *Note added in the 2nd revised edition:* Another Appendix, included in the first edition of this report, on the multi-layer depth–dose code EDMULT for electron beams has been deleted, because a separate report on it was published later (RCOP Tech. Rep. 8). Other modifications of the present edition are the deletion of figures and the corrections of minor errors. A review of the later work by Tabata and his coworkers will be given in another issue of Institute for Data Evaluation and Analysis Technical Report.

CONTENTS

I. INTRODUCTION

During the last twenty years, we have been working on the problems concerning the passage of fast electrons through matter at the Radiation Center of Osaka Prefecture (RCOP). Shigeru Okabe, now professor at Fukui University, initially suggested us the problems, guided earlier work, and was the coauthor of many of our publications. Our earlier studies were experimental, and later ones were computational. In this report, some important results of our work are reviewed along with related topics.[4] Problems to be solved and future plans of our research are also briefly described. Section II treats our main experimental work, and the computational work is described in Sec. III. In Appendix, the work on the interpolation formulas for the continuous slowing-down approximation (CSDA) range is reviewed.

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[4] In this review, we lay emphasis on the historical trace of the interrelation between our work and that of other authors with the hope of providing, as a byproduct, a hint on the nature of the development of scientific research. Review articles written in Japanese on the passage of electrons through matter have been published (Ta72a, Ok76, Na82). A compilation of the relevant data and references that appeared in the period from 1968 to 1978 has been given by Haruo Sugiyama of the Electrotechnical Laboratory (Su79). The theoretical work up to 1972 has been reviewed by Hans-Wolf Thümmel of the Academy of Sciences of the German Democratic Republic (Th74).

II. EXPERIMENTAL WORK

We started experimental study on electron penetration by the use of a linear electron accelerator in the early 1960's. In those days the data on the passage of electrons through thick layers of matter were scarce for energies above 1 MeV. Part of the present title "The passage of fast electrons through matter" is the title of the review article in Handbuch der Physik written by R. D. Birkhoff of the Oak Ridge National Laboratory (Bi58). The last section of the article is entitled "Electron penetration through thick layers." We had an aim of revising the contents of that section through our research.

Two experiments made or in progress at that time served as a spur to us; one was the measurement of the backscattering coefficients of electrons of energies 1–3 MeV made by Kenneth Wright and John Trump of the Massachusetts Institute of Technology (MIT) (Wr62), and the other was Yohta Nakai's work (Na63) on the energy dissipation of electrons of energies 1–2 MeV in progress at the Japanese Association for Radiation Research on Polymers.

1. Measurement of Backscattering Coefficients

We made a start by the measurement of the backscattering coefficients[5] of the electrons of energies from 3 to 14 MeV (early accounts are given in refs. Ok63 and Ok65a). The experiment on the backscattering of 1.75-MeV electrons done by H. Frank of Göttingen University (Fr59) was a good example for our work.[6] When our experiment was still at a preliminary

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[5] Okabe thought of this theme from the work on the backscattering of gamma rays done by a friend of his, Tomonori Hyodo of Kyoto University (Hy62). Our study on the backscattering of electrons in turn lead ourselves later to the study on the backscattering of light ions (Ta81, Ta83). In the latter case, Okabe's suggestion that there were many problems to be solved in the plasma-wall interaction also gave us a clue.

[6] In 1958 G. Breitling of Tübingen University (Br58) reported the experimental values of the backscattering coefficient for energies from 2.5 to 15.8 MeV. However, he used a divergent electron beam after passing through a scatterer foil, and the result lacked the significance as basic physical data.

stage, Dietlich Harder and H. Ferbert of Würzburg University (Ha64) reported the backscattering coefficients for energies from 8 to 22 MeV.

It was lucky that our method of measurement was different from theirs. We were measuring the backscattering coefficients differential in scattering angle using the X-ray compensation type of ionization chamber developed by R. J. Van de Graaff, W. W. Buechner and H. Feshbach of the MIT (Va46), and considered it worth to continue our experiment.

At the time our experiment was almost completed, Ralph W. Dressel of New Mexico State University reported the backscattering coefficients for energies from 0.5 to 10 MeV (Dr66). His results were appreciably higher than the results obtained by the majority of previous authors by a factor of about two, and he discussed that the discrepancy might be attributed to errors in the previous work. On the basis of our results, we could timely report (Ta67a) that the cause of the discrepancy might be in Dressel's work. This was confirmed by D. Harder and L. Metzger at Würzburg (Ha68) and by P. J. Ebert, A. F. Lauzon and E. M. Lent of the Lawrence Radiation Laboratory (Eb69). Dressel himself carefully checked his experiment later, and found that his error was caused by the peripheral halo of electrons that was accompanying the main beam and passed unnoticed (Dr68).

We learned from this incident that much care should be taken in the seemingly simple task of monitoring the electron beam current.[7] This is a rather old story around the mid 1960's.

2. Measurement of Charge-Deposition Distributions

Our next plan was to study the range of electrons.[8] Harder again went ahead, and reported with G. Poschet the measurement of the transmission

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[7] From our data, we also pointed out minor errors in the data of Harder and Ferbert at the lowest energy of their experiment. The cause of these errors was the dependence of the monitoring system on dose rate. Harder and Metzger (Ha68) published corrected values. Okabe and the present authors, sometimes with Kunihiko Tsumori, made also studies on monitoring methods for various parameters of the electron beam. An account of earlier part of these studies is given in the review article written by Okabe (Ok67).

curves for the electrons of energies from 4 to 30 MeV (Ha67). We took a different approach of measuring the charge-deposition distribution using a thin collector moved through the absorber thickness (Ta71b). This time the work on the same theme done by Bernhard Gross and K. A. Wright of the MIT using 3-MeV electrons was a good example (Gr59). From the differential charge-deposition distribution observed, one can obtain the integral charge-absorption curve and the extrapolated range of electrons as defined for the semi-infinite medium. The values obtained of the extrapolated range (Ta71c) were in good agreement with the values of Harder's group, the latter representing the extrapolated range as defined for the slab geometry.

This indicates that the extrapolated range can be defined independently of the two different configurations of the absorber. At almost the same time as we made this experiment, Harder and H. J. Schulz (Ha72) extended the measurement of the extrapolated range up to 62 MeV. Their results will again be referred to in the next section.

Until the late 1960's, the method of computer simulation had not been widely used for solving the problems of electron penetration. Before publishing our results of charge-deposition experiment, one of us (T. T.) had a chance[9] to see L. V. Spencer of the National Bureau of Standards (NBS), who developed the moments method to solve the problem of electron penetration (Sp55). T. T. showed our data to Spencer, and asked him whether it was possible to compute charge-deposition distributions with the moments method by taking the effect of radiative process into account. Spencer answered that we should compare our data with the results obtainable by the Monte Carlo code ETRAN just developed by Martin Berger and Stephen Seltzer of his institute (Rs69). We sent our data to Berger, and he kindly sent us Monte Carlo results to be compared with our data.

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[8] First we tried to determine the maximum range of electrons by measuring the tail of the transmission curve and analyzing it by the *n*th-power method. However, it was found that the definition of the maximum range associated with this method was not generally valid (Ta69).

[9] It was on the occasion of Spencer's visit to Kyoto after the 12th International Congress on Radiology held in Tokyo in 1969. He was awarded the Gray medal at the Congress.

Agreement between the experimental and the Monte Carlo results was good except a slight discrepancy for the beryllium absorber. For beryllium the calculated distribution has lower values at the left of the peak, and higher values at the top and the right of the peak, than the measured distribution, thus showing higher penetrability of electrons.

In his reply to our letter, Berger (Be70) has given four possibilities that could cause errors in the calculation:

(1) The Landau energy-loss straggling distribution might not be sufficiently accurate.
(2) The Monte Carlo model might in some manner be inadequate.
(3) The step-size chosen might not have been fine enough.
(4) The treatment of multiple scattering deflections in electron-electron collisions could be at fault.

H. Roos, P. Drepper and D. Harder (Ro74) reported angular distributions of electrons transmitted through foils in the transition region from multiple scattering to complete diffusion,[10] and compared the results for 20-MeV electrons passing through aluminum slabs with Berger's Monte Carlo calculation. They found that at small angles the calculation yielded higher values than the experiment. There is the possibility that this discrepancy was caused by the same fault in the calculation as was responsible for the discrepancy in the charge-deposition distribution in beryllium, though discrepancy was unappreciable in the charge-deposition distribution in aluminum. Roos et al. considered that the fault was in the use of Thomas–Fermi statistical model of the atom in Gert Molière's treatment of single scattering (Mo47). This is another possibility to be added to Berger's four items aforementioned.[11]

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[10] We made a similar experiment (Ta67b), but have not accomplished the refinement of the work we planned.

[11] J. A. Lonergan, C. P. Jupiter and G. Merkel of Gulf General Atomic Incorporated also reported discrepancies between experimental and Monte Carlo results in the following cases: (1) the number of 4-MeV electrons transmitted through an aluminum slab and (2) the energy spectrum of 8-MeV electrons transmitted through a beryllium slab (see the discussion in ref. Ta71b).

Later, measurement of charge-deposition distributions was also made by J. Van Dyk and J. C. F. MacDonald of Victoria Hospital (Va72a, Va72b) for the electrons of energies from 10 to 32 MeV incident on water. Takehiro Nishidai et al. of Kyoto University made the experimental study on the charge-deposition distributions of the electrons of energies from 4 to 32 MeV in water and saline solution (a summary of this work is given in ref. Ni76; a graphical presentation of one of the results, in ref. Ni78).

3. Remaining Problems

To get further knowledge on the cause of the discrepancies between the experimental and Monte Carlo results, Okabe suggested us to make experiment of multiple scattering for the liquid-hydrogen or liquid-helium target, for which the deviation of the Thomas–Fermi model from the true scattering potential was expected to be the largest. However, it was found to be extremely difficult to carry out this experiment with our present facility. Several years ago, we changed our plan for using beryllium and carbon targets, but since then we have not been able to have time for the experiment. Concerning the same topic, we should note the work published by B. W. Mayes et al. of the University of Houston and Rice University (Ma74). They measured the small angle multiple scattering of pions, and used all previous multiple-scattering data for electrons and protons as well as their own pion data to reevaluate the empirical term in Molière's screening function. The original Molière's function is given by

$$f\left(\alpha^2\right) = 1.13 + 3.76\alpha^2, \qquad (1)$$

where

$$\alpha = zZe^2/\hbar v, \qquad (2)$$

The symbol ze represents the charge of the projectile, Ze is the charge of the target nucleus, $\hbar$ is the Planck constant divided by 2π, and v is the velocity of the projectile. The new function obtained by Mayes et al. is given by

$$f\left(\alpha^{2}\right) = 0.59 + 3.44\alpha^{2}. \tag{3}$$

In determining the constants in eq. (3), Mayes et al. ignored the beryllium data of the time-honored experiment by Alfred Hanson et al. of the University of Illinois (Ha51). It might be worthwhile experimentally to check the validity of eq. (3) at small values of α, and also to examine how much the discrepancies found between the experiments and the calculations on the charge-deposition distribution and the multiple scattering can be reduced by the use of the new screening function.

III. COMPUTATIONAL WORK

We began to concentrate more in computational work than in experimental one when we joined the research program (Ok74) of the Committee on Electron Dose Measurement, Radiation Branch, the Japan Society of Applied Physics. Okabe was the chairman of this Committee. Our computational work is classified into three categories:

1. The formulation of empirical or semiempirical formulas for experimental parameters describing the penetration of electrons.
2. The formulation of approximate analytical expressions for theoretical parameters related to the passage of electrons.
3. The formulation of semiempirical algorithms for depth–dose distributions of electrons in matter.

Important results obtained in the work of the first category are the empirical formula for the backscattering coefficient and the semiempirical formula for the extrapolated range.[12]

1. An Empirical Formula for the Backscattering Coefficient

Our formula for the backscattering coefficient η (Ta71a) is expressed as a function of the incident energy and the atomic number Z of the target. Electrons are assumed to be normally incident on the effectively semi-infinite target. The lower energy limit to the region of validity of the formula is about 50 keV for low Z targets and about 200 keV for high Z targets. We now consider extending the formula to the region below these

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[12] Generalization of the formula for the extrapolated range was a theme we had kept in mind since we had read the work of S. P. Khare and Y. P. Varshni of Allahabad University (Kh61). They modified Flammersfeld range–energy relation for electrons in aluminum to extend the region of validity to lower energies.

limits, where η as a function of incident energy shows different behaviors depending on Z. The functional form of our formula is given by[13]:

$$\eta = a_1 \big/ \left(1 + a_2 \tau^{a_3}\right), \tag{4}$$

where the symbols a_i ($i = 1, 2, 3$) denote constants for a given target material, and τ is the incident energy expressed in units of the rest energy of the electron. This form has been determined from the fact that the dependence of η on energy shows a shape of a logistic curve on a semi-logarithmic plot in the energy region above the lower limits aforementioned.

In the energy region between 10 and 100 keV, the dependence of η on incident energy is rather weak, and eq. (4) can be approximated as

$$\begin{aligned} \eta &= a_1 \\ &= 1.15 \exp\left(-8.35 Z^{-0.525}\right) . \end{aligned} \tag{5}$$

Using a simple model of electron transmission, Pierre Verdier and Floreal Arnal of the Centre National de la Recherche Scientifique (Ve69) have derived a semiempirical formula valid in this energy region; their formula is given by

$$\begin{aligned} \eta &= 2^{-9.8/\sqrt{Z}} \\ &= \exp\left(-6.8 Z^{-0.5}\right) . \end{aligned} \tag{6}$$

This is of the same functional form as eq. (5), and the values of constants are also close to those in eq. (5). At high energies where the relation $a_2 \tau^{a_3} >> 1$ is valid, eq. (4) is approximated as

$$\eta = a_1 \big/ \left(a_2 \tau^{a_3}\right), \tag{7}$$

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[13] V. A. Kuzminikh et al. of the Tomsk Polytechnical Institute (Ku74, Ku75) have reported the formula for the backscattering coefficient of positrons by fitting eq. (4) to the data obtained by the segment model computation.

where a_3 takes on values between 0.823 (for $Z = 6$) and 1.51 (for $Z = 92$). Previously we proposed the following equation for energies above about 10 MeV (Ok65b):

$$\eta = 0.022(Z/\tau)^{1.2}. \tag{8}$$

This is similar to eq. (7) in its dependence on τ. Therefore, eq. (4) approximately contains both eqs. (6) and (8) as limiting cases, and can be regarded as a unification of these equations.

The highest-energy data used in determining the adjustable parameters of the backscattering formula are those of Harder and Ferbert (Ha67) at 22 MeV. John Pruitt of the NBS (Pr72) measured the backscattering loss of electrons from a Faraday cup for energies from 20 to 120 MeV, and found that the dependence of the fractional loss upon energy was well represented by our formula. This indicates that our formula is valid at least up to about 120 MeV.

G. B. Radzievsky of the Institute of Biophysics in Moscow (Ra81) has proposed a generalization of the similarity principle in electron penetration originally found by Harder (Ha70). The former author has defined the isoline on the coordinate plane spanned by atomic numbers and incident energies as the line along which the necessary conditions for the similarity of scattered-electron fields are fulfilled, and has shown that a family of isolines can be obtained from the data or the empirical formula for the backscattering coefficient. Therefore, one can formulate an empirical formula for a parameter describing electron penetration as a function of atomic number and incident energy when we know:

(1) the dependence of this parameter on atomic number at a single energy,
(2) the inverse function of the backscattering coefficient solved for incident energy,
(3) the inverse function of the backscattering coefficient solved for atomic number,

(4) the scaling length for the similarity (this is close to, but not always equal to, the CSDA range).

Among these, the item (3) cannot analytically be obtained from our empirical formula. Therefore, we have a plan to formulate approximate analytic expression for this function.

2. A Semiempirical Formula for the Extrapolated Range

Our semiempirical formula for the extrapolated range R_{ex} of electrons (Ta72b) is given as a function of the incident energy as well as the atomic number and the atomic weight of the absorber. The functional form to express the dependence of R_{ex} on incident energy has been determined by integrating an approximate expression for the inverse of the stopping power, and is given by

$$R_{\mathrm{ex}} = b_1\left[\left(1/b_2\right)\ln\left(1+b_2\tau\right) - b_3\tau\Big/\left(1+b_4\tau^{b_5}\right)\right], \tag{9}$$

where b_5 is an additional parameter introduced to improve the fit and takes on values close to unity. At low energies the form of this formula approaches the following equation developed by K.-H. Weber of VEB Vakutronik WIB (We64) to express both the CSDA range R_0 and the extrapolated range R_{ex} in aluminum:

$$R = c_1\tau\left[1 - c_2\Big/\left(1+c_3\tau\right)\right]. \tag{10}$$

where R denotes either R_0 or R_{ex}. At high energies the form of eq. (9) approaches the following formula for R_0 derived by H. W. Koch and J. W. Wyckoff of the NBS (Ko58):

$$R_0 = \left(B_1 A/Z^2\right)\ln\left(1+B_2 Z\tau\right), \tag{11}$$

where A is the atomic weight of the absorber material. Thus eq. (9) is a unification of eqs. (10) and (11).

Our formula is valid down to about 0.3 keV. The highest-energy data used are again those of Harder's group, and are at the energy of 30 MeV.

Harder and Schulz (Ha72), as mentioned before, reported extrapolated ranges at higher energies determined from the measurement of depth–dose curves. Their results at 46 and 62 MeV are higher for copper and lead absorbers than the values expected from the extrapolation of the earlier results of Harder and Poschet.

A. O. Zaimidoroga et al. of the Joint Institute for the Nuclear Research in the U.S.S.R. (Za66) measured the cascade curves for high-energy electrons in lead. The cascade curve represents the number of electrons and positrons as a function of depth. The extrapolated range determined from the cascade curve at 45 MeV is favorable to the results of Harder and Schulz, and these authors considered that there were two possible reasons for the systematic deviation between their data and those of Harder and Poschet:

(1) The counter used by Harder and Poschet registered only a single count for an electron–positron pair.

(2) The absorber configuration used by Harder and Poschet was the finite slab, while Harder and Schulz employed the effectively semi-infinite medium.

We consider that there is a third and the most probable reason for the discrepancy; the energy-deposition curve might yield different values of the extrapolated range than the number-transmission curve. The agreement of the results of Harder and Schulz with the data of Zaimidoroga et al. is apparently a puzzle for this reason to be true. However, the determination of the extrapolated range from the cascade curve of Zaimidoroga et al. is considered to have a large ambiguity.

One of us (T. T.) met Harder on the occasion of the 6th International Congress of Radiation Research held in Tokyo in 1979. On that occasion, however, the former did not think of discussing the problem of this discrepancy; they talked on a slightly different problem. T. T. told Harder that one should employ the method of measuring the charge deposition in determining the extrapolated range of high-energy electrons. Harder answered that the method to measure the extrapolated range (also called practical range) should be practical and that the use of the ionization chamber was the most practical. T. T. argued that the measurement of charge deposition was simple and rather insensitive to the bremsstrahlung background. To settle these problems on the extrapolated range of high-energy electrons, further study might be necessary.

3. Empirical Formulas for Other Parameters

Besides the formulas for the backscattering coefficient and the extrapolated range, we have developed the empirical formulas for the following parameters:

1. The projected range (Ta68, Ta71c).
2. The projected-range distribution (essentially the same as the charge-deposition distribution except the effect of secondary electrons) (Ta71d).
3. The most probable range (Ta71e).
4. The average energy loss fraction of backscattered electrons (Ta71f, Ta72c).
5. The incident energy as a function of the corresponding extrapolated range (Ta72b).
6. The average residual energy at a given depth of the absorber (Ta73, Ta74).
7. The number transmission coefficient; normal incidence (Ta75b), oblique incidence for the Al absorber (Ta76b).
8. The backscattering coefficient for the point isotropic source (Ta79b).

4. Approximate Expressions for Theoretical Parameters

In the second category of our computational work, we have developed approximate expressions for the following parameters:

1. The CSDA range (It71).

(2) The energy dissipation in the infinite medium by electrons from the plane perpendicular source (Ta72d, Ta72e) (an interpolation formula for the results of the moments-method computation published in ref. Sp59).

(3) The energy deposition of electrons obliquely incident on the semi-infinite medium (Ta75a) (an interpolation formula for the results of ETRAN tabulated in ref. Wa71).

(4) The parameter B in Molière's theory of multiple scattering (Ta76a).

(5) The quantity $\phi_{\mathrm{rad}}/\overline{\phi}$ proportional to the radiative energy loss divided by the total energy of the incident electron (Ta77).

(6) The empirical correction factor to be applied to the Born-approximation result for the bremsstrahlung cross section (Ta77).

(7) The function $\cos\gamma$ in the formula for Mott to Rutherford single scattering ratio (Ta78).

(8) Landau's distribution function for the ionization energy loss of electrons (Ta79a).

The approximate expressions obtained are considered useful for economizing the computation on the problems of electron penetration. Approximations for the CSDA range have been proposed also by a number of authors; a brief review of related work is given in Appendix.

5. Algorithms for the Depth–Dose Distribution

The results of the third category of our computational work are two algorithms for the depth–dose distribution of the plane-parallel electron beam normally incident (1) on a semi-infinite absorber consisting of a single material (Ta74) and (2) on a two- or three-layer slab absorber (Ta81a). The first algorithm[14] for a single material is a refinement of the algorithm developed by E. J. Kobetich and Robert Katz of the University of Nebraska (Ko69). In making the refinement, the empirical formulas described in subsections 1–3 were used. The energy region in which this algorithm is valid is limited by the regions of validity of the empirical formulas used, and has been estimated to extend from 0.1 to 20 MeV. The algorithm can be incorporated in the expression for the depth–dose distribution of the electron beam with a finite width or cross section (see ref. Ma83 and references cited there).[15]

The algorithm for the multi-layer absorber is based on a simple model of electron penetration across the interface, and the algorithm for a single material is repeatedly used. The depth–dose distributions obtained by the algorithm for the multi-layer absorber were compared (Ta81a) with the results of experiment and computer simulation both reported by Harvey Eisen, Marvin Rosenstein and Joseph Silverman of the University of Maryland (Ei72).[16] We have published a FORTRAN code for this algorithm in *RCOP Technical Report* (Ta81b). Recently we have made minor revisions of the code, and placed it in the computer code collection at the Radiation Shielding Information Center, Oak Ridge National Laboratory (Rs82).

The algorithm in its revised form uses only two adjustable parameters besides those contained in component formulas. One parameter is used in a function that makes interpolation of the backscattering coefficient between normal and isotropic incidence; the other is used to set an upper

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[14] This algorithm is given in the form of a FORTRAN subprogram. The name of the subprogram is EDEPOS.

[15] At the Symposium on Model Computation of Doses in Multidimensional Media held at the Kyoto University Research Reactor Institute in 1976, Kiyomitsu Kawachi of the National Institute of Radiological Sciences delivered a lecture on his work published the year before (Ka75), in which he derived an expression for the two-dimensional dose distribution of therapeutical electron beams using the age diffusion equation. Being nominated as a commenter on his lecture, one of us (T. T.) remarked that the term in his expression describing the dose distribution along the depth could be interpreted as the depth–dose distribution d_{SIM} of the plane-parallel electron beam normally incident on the semi-infinite absorber and that our algorithm could be substituted for this term to extend the region of validity of his result. Moshen A. Mandour, Fridtjof Nüsslin and Dietlich Harder (Ma83) have pointed out the importance, in electron dose planning, of the knowledge on d_{SIM} from the fact that it appears in the expression for the three-dimensional dose distribution of the electron beam with the rectangular or the circular cross section.

[16] Further comparison with the Monte Carlo results reported by M. J. Kniedler and J. Silverman (Kn68, Kn69) and the experimental and Monte Carlo results reported by Grant Lockwood, Glenn Miller and John Halbleib of Sandia Laboratories (Lo76) are given

limit to the solution of an equivalent incident energy required in applying a principle of equivalence. The principle of equivalence we have employed was originally proposed by Ryuichi Tanaka, Keiichi Yotsumoto and Yoshiteru Nakamura of the Japan Atomic Energy Research Institute (Ta71d) as a basis of a simple means to calculate the depth–dose distribution in the second layer of the two-layer absorber. The success of the algorithm owes much to the validity of this principle.

We have a plan to extend the algorithm to include the case of the electron beam incident with an arbitrary obliquity. For this purpose it is necessary to generalize the empirical formulas for the backscattering and transmission coefficients so as to include the dependence on angle θ of incidence. Though a number of formulas have been proposed to express the dependence of the backscattering coefficient on θ for different regions of incident energy (Ko65, Ar69, Da75, Ku75, Ra78, Ne80; see also the recent review article Ni82 by H. Niedlig of Technical University Berlin), no attempt has been made to develop a single formula covering a wide region of energy. For the transmission coefficient of obliquely incident electrons, we have given a formula that covers a wide energy range (Ta76b), but it is applicable only to the absorber of aluminum. We still have many problems to solve for facilitating electron dose planning.

IV. CONCLUDING REMARKS

During these twenty years, the main interest of the work on the passage of fast electrons through matter has shifted from the field of physics to the fields of applied physics and engineering.[17] In this niche of research, we have seen a steady increase of knowledge, and there were monumental

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in refs. Ta81a and Ta81c. The results of the latter authors have been compiled in ref. Lo80 together with additional data.

[17] As far as we have noticed, only two papers (Ba81, Hu82) have been published in *The Physical Review* since 1972 concerning the passage of fast electrons through matter. Much effort in this area has recently been concentrated on the dose planning of therapeutic electron beams (see, for example, refs. Br76, Br80, Br81 and Be82).

achievements such as a series of studies by Harder's group[18] including his finding of the similarity principle, and the Monte Carlo code ETRAN[19] and the stopping power and range tables (a new edition: Be83) of Berger and Seltzer, to enumerate a few. We consider that our work, intertwined with that of other authors, has also served to give conveniences for electron beam applications.

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[18] Earlier work has been summarized in ref. Ha65.

[19] A code applicable to multi-slab geometries and containing ETRAN as a subset has been developed by John Halbleib and W. H. Vandevender of Sandia Laboratories (Ha74).

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APPENDIX. APPROXIMATIONS TO THE CSDA RANGE

In this appendix, approximate expressions for the CSDA range R_0 of electrons are reviewed, and a new formula is suggested.

In recent work, adjustable parameters in the interpolation formula for R_0 have been determined by using either the tabulation of Berger and Seltzer (Be64) or that of Pages et al. (Pa72). In the following, therefore, the accuracy of the formulas reviewed will be stated as "within . . .% of table Be64," when the table of Berger and Seltzer was used, or "within . . .% of table Pa72," when the table of Pages et al. was used.

Wilson (Wi50, Wi51) derived a simple analytic expression for R_0 of high-energy electrons. It is written as

$$R_0 = (\ln 2)\chi_0 \ln\left[1 + (\tau/\tau_c)\ln 2\right], \tag{A.1}$$

where τ is the incident energy in units of the rest energy of the electron, and χ_0 and τ_c are the radiation length (in the same unit as R_0) and the critical energy (in the same unit as τ) of absorber material, respectively.

The formula of Koch and Wyckoff (Ko58) and that of Weber (We64) have been shortly described in Sec. III.2. The former authors' formula is essentially the same as eq. (A.1).

Vzorov (Vz69) has proposed an interpolation formula of the following form:

$$R_0 = a(\ln T)^b, \tag{A.2}$$

where T is the incident energy in MeV, and values of a and b are tabulated for various materials. This expression is accurate within about 5–8% of table Be64 in the energy region from 20 to 1000 MeV. He has also given a relation to express these parameters as a function of the atomic number Z of absorber material:

$$a^{-1} \text{ or } b = \alpha \exp\left(-\beta Z\right) + \gamma Z^{\delta}. \tag{A.3}$$

Watts and Burrell (Wa71) have given the following parametric fit to R_0 in aluminum:

$$R_0 \left(\mathrm{g/cm^2}\right) = \left(1.33 - 0.019T\right)\left[\left(0.2713T^2 + 0.0121\right)^{1/2} - 0.11\right]. \tag{A.4}$$

This formula is accurate within about 2% of table Be64 at energies greater than 0.3 MeV, and within 5% between 0.2 and 0.3 MeV. Shreve and Lonergan (Sh71) have suggested that for other elements the right-hand side of eq. (A.4) can be multiplied by $2.08Z/A$ (A is the atomic weight of absorber material) to obtain R_0 accurate within about 16% of table Be64 in the energy region from 0.3 to 10 MeV.

Ito, Tabata and Okabe (It71) have reported a formula of the form:

$$R_0 = a_0\tau\left(1 + a_1/\tau\right)\Big/\left\{\left[1 + \left(\alpha\tau\right)^{\beta}\right]\left(1 + a_2/\tau\right)\right\}, \tag{A.5}$$

where a_i (i = 0, 1, 2), α and β are adjustable parameters depending on material. When these parameters are expressed as a function of Z and A by using a total of thirteen adjustable parameters, eq. (A.5) gives values of R_0 accurate within 5% of table Be64 for arbitrary elemental material in the energy region from 10 keV to 1000 MeV.

On the basis of a simple atomic model, Kanaya and Okayama (Ka72) have obtained the following semiempirical formula valid in the energy region from 10 keV to 1 MeV:

$$R_0 \left(\mathrm{g/cm^2}\right) = \frac{2.76\times10^{-11}\left(A/Z^{8/9}\right)T^{5/3}\left(1 + 0.978\times10^{6}T\right)^{5/3}}{\left(1 + 1.957\times10^{-6}T\right)^{4/3}}. \tag{A.6}$$

Mukoyama (Mu76) has proposed a modified Wilson's formula in which Knasel's formula (Kn70) is used for the radiation length and a correction factor is multiplied to the original Wilson's formula. The correction factor $F\left(T\right)$ is given by

$$F(T) = 1.5 - 1.3\exp(-2T) \; (T \text{ in MeV}). \tag{A.7}$$

Mukoyama's formula is accurate within 10% of table Pa72 in the energy region above 70 keV for silicon, above 60 keV for germanium, and above 55 keV for NaI. Gupta and Gupta (Gu81) incorporated the expression of dependence on Z into the correction factor; their formula is accurate within 7% of table Pa72 for various materials in the energy region from 50 keV to 100 MeV.

Matthews (Ma80) has given the values of the correction factor $F(T)$ for the aluminum absorber in tabular form, and pointed out that the application of the same correction factor for other materials gives values of R_0 accurate within 5% of table Pa72 at energies above 10 MeV, and 15% above 10 keV. The present authors have found that Matthews's values of the correction factor can be fitted within the minimax error (the error of the best approximation in Tchebychev's sense) of 2.0% by

$$F(\tau) = 2.4986 \Big/ \left(1/\tau^{0.8895} + 1.3215\tau^{0.4276}\right). \tag{A.8}$$

We are presently trying to fit the equation of the following form as well as its modifications to the new values of R_0 reported by Berger and Seltzer (Be83):

$$R_0 = c_1\left[\ln(1 + c_2\tau)/c_2 + c_3\ln(1 + c_4\tau) - c_5\tau/(1 + c_4\tau)\right]. \tag{A.9}$$

The values of c_i (i = 1, 2, ..., 5) determined and the minimax error attained are given in Table A.I for the case of the aluminum absorber. The form of eq. (A.9) is obtained by assuming the following form for the stopping power S:

$$S = \left\{k_1 \Big/ \left[1 - k_2 \Big/ (1 + k_3\tau)^2\right]\right\}(1 + k_4\tau). \tag{A.10}$$

Putting eq. (A.10) into the relation

$$R_0 = \int_0^\tau (1/S)\,\mathrm{d}\tau, \tag{A.11}$$

we obtain eq. (A.9) with

$$c_1 = K \Big/ \left[k_1 \left(k_3 - k_4 \right) \right], \tag{A.12}$$

$$c_2 = k_4 \tag{A.13}$$

$$c_3 = k_2 k_4 \Big/ \left[K \left(k_3 - k_4 \right) \right] \tag{A.14}$$

$$c_4 = k_3 \tag{A.15}$$

$$c_5 = k_2 k_3 / K, \tag{A.16}$$

where K is given by

$$K = \left[k_3^2 - 2k_3 k_4 + \left(1 - k_2\right) k_4^2 \right] \Big/ \left(k_3 - k_4 \right). \tag{A.17}$$

From this derivation of eq. (A.9), one can expect that the dependence of the parameters c_i (some of them at least) on absorber material is estimable from theoretical relations.

Table A. I. Values of parameters in eq. (A.9) for the case of the aluminum absorber. The minimax error attained of the approximation is also given.

Parameter	Value
c_1	0.3521
c_2	0.014999
c_3	0.11454
c_4	1.630
c_5	1.1674
Minimax error = 2.0%	

Note added after completion of the manuscript for the first edition: Recently J. Kalef-Ezra and Y. S. Horowitz of Ben Gurion University of the Negev, Israel, and J. M. Mack of the University of California reported a work on the backscattering of electrons of energies below 1 MeV incident on low atomic-number targets. They studied a modification of Everhart's formula for the backscattering coefficient in the region of incident energy T from 1 to 50 keV, and also the validity of existing empirical formulas for η as a function of angle of incidence ($10\text{ keV} \le T \le 100\text{ keV}$, Seidel–Darlington formula; $T \ge 100\text{ keV}$, Kuzminikh–Vorobiev formula) [*Nuclear Instruments & Methods in Physics Research* **195**, 587 (1982)].

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